WEAKLY NONLINEAR OSCILLATIONS IN BULK CRYSTALLIZATION

Yu. A. Buevich, V. V. Mansurov, and
I. A. Natalukha

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The example of homogeneous crystallization is used to examine stationary almost-harmonic oscillations due to instability in the formation and growth of new-phase elements in a matrix.

In [1] it has been shown that nucleation in a metastable medium under stationary external conditions is often unstable: small perturbations in the nucleation rate under certain circumstances increase exponentially with time. The nonlinearities stabilize this, which leads to the establishment of a specific self-excited oscillation. The effect is evidently very important in crystallization from supersaturated solutions or supercooled liquids in a crystallizer in which the solid particles are continuously removed [2, 3]. The tendency for various instabilities to occur in such systems has been pointed out for example in [4, 5].

According to [1], the main reason for instability lies in the avalanche formation of critical nuclei when a certain supersaturation or supercooling is attained. The growing crystals eliminate this. If the nucleation rate varies sharply with these factors, the production of fresh nuclei is greatly retarded, and the supersaturation or supercooling may decrease to values lower than those corresponding to a constant nucleation rate under the given external conditions. As the crystals are removed, the concentration or the cooling will increase, which at first is virtually not compensated by fresh nucleation, and as a result the supersaturation or supercooling again attains the values corresponding to avalanche nucleation, after which the process repeats. This leads to periodic oscillations even under stationary conditions, as has been repeatedly observed [4-6].

Similar instabilities and oscillations in nucleation are characteristic of processes such as homogeneous or heterogeneous boiling, which has been discussed in [1]. Oscillatory phenomena in boiling have also been quite frequently observed [7]. However, there was a difference from [1], where the main attention was given to large-amplitude oscillations such as are particularly characteristic of boiling (shock or explosive boiling), which may be described as discontinuous relaxation oscillations, in that in this paper studies were made on the weakly nonlinear almost harmonic oscillations arising with only slight supercriticality under conditions specific for homogeneous—crystallization systems.

These oscillations are extremely important in a purely applied respect, not only in relation to eliminating the instability if this is undesirable but also in order to make effective use of it to improve parameters such as the overall yield of pure product, the mean crystal size, and so on, as well as to accelerate heat transfer.

For definiteness, we consider crystallization from supersaturated solutions, although all the arguments and expressions can be transferred without particular difficulty to other situations such as supercooling in solutions or melts. The physical characteristics are taken as constant, while the bulk crystal concentration is taken as small, while we neglect the tendency of the nucleation rate to its stationary value because the characteristic relaxation time is usually less by several orders of magnitude than the time scale of the oscillation.

We follow [1] in describing mass crystallization via a balance equation for the solute and a kinetic equation for the distribution $f(t, r)$ of the crystals by radius r as normalized to the numerical concentration. We assume that the mass input to the system per unit volume and unit time is constant at Q, and the first of these equations is then written as

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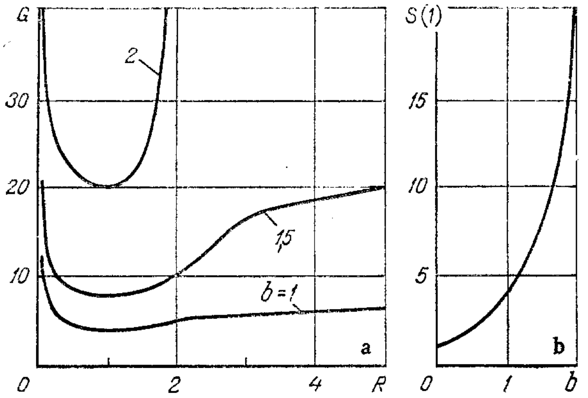

Fig. 1. Neutral-stability characteristics: a) neutral-stability curves in the plane of the parameters (R, $G = J'u_s/J$); b) dependence of the minimal value of S in (1) for S(R) on b.

$$\frac{dc}{dt} = Q - \rho \int_{r_*}^{\infty} \frac{dv}{dt} f(t, r)\,dr, \quad v = \frac{4\pi}{3} r^3. \tag{1}$$

The assumption is made in deriving (1) that the system is homogeneous, which corresponds to ideal mixing, which usually describes real situations well [2, 3]. In principle, Q can depend in any way on the supersaturation.

The kinetic equation and the boundary condition for it are written as

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial r}\left(\frac{\partial r}{\partial t} f\right) + \gamma f = 0, \quad \left.\frac{dr}{dt} f\right|_{r=r_*} = J\left(\frac{c-c_0}{c_0}\right). \tag{2}$$

Here γ is considered as an effective parameter characterizing the rate of crystal removal, while the formation rate for critical nuclei of radius r_* is taken as dependent only on the relative supersaturation (the concentration c_0 corresponds to thermodynamic equilibrium). The assumption that γ is independent of r represents an idealization, but it is usually employed in theoretical studies; as a whole, it reflects the situation quite well. Such studies also usually involve the assumption that the process is isothermal. It will be evident from the subsequent analysis that abandoning both of these simplifications does not lead to essential difficulties.

The growth rate dr/dt is usually taken as a power-law dependence on the radius with exponent $\alpha < 1$, the value being determined by the detailed growth condition [11, 12]. Limiting situations occur under kinetic conditions, where $\alpha = 0$ (this simplest condition is considered in [8-10]), and under diffusion-limited conditions, where the growth rate is limited by the rate of diffusion of the solute to the crystal and $dr/dt = (D/\rho r)(c - c_0)$, i.e., $\alpha = -1$ [13]. In what follows, we put $\alpha = 1 - b$, $b > 0$, and introduce the variables

$$\theta = \gamma t, \quad u = \frac{c - c_0}{c_0}, \quad \frac{dr}{dt} = \frac{\beta c_0 u}{r^{b-1}}, \tag{3}$$

where the kinetic coefficient β can have any dependence on the relative supersaturation.

Then system (1), (2) can [1] be transformed to a single evolutionary integrodifferential equation, which in the variables of (3) takes the form

$$\gamma \frac{du}{d\theta} = \frac{Q}{c_0} - \frac{4\pi\rho u}{b u_0 c_0}\left(\frac{b\beta c_0 u_0}{\gamma}\right)^{3/b} \int_0^{\infty} J\,[u(\theta - \tau)]\,\tau^{-1+3/b} e^{-\tau} d\tau, \tag{4}$$

where $u_0 = \langle u \rangle$, i.e., the time average of u(t). We have derived (4) on the assumption that the time scale T satisfies $\gamma T \gg 1$, while the mean radius of the spherical crystals is much greater than r_* [1].

The stability of the stationary state with a constant crystal formation rate can be perturbed in mild or severe fashions. The second case has been examined in [1] in the discontinuous—oscillation approximation for homogeneous boiling. In the first case, we consider slightly supercritical conditions in order to elucidate the perturbation evolution, i.e., the instability region near the neutral-stability curve. Then $u(\theta)$ can be put as $u_s(1 + \xi)$, where $|\xi(\theta)| \ll 1$, while the constant u_s represents the dimensionless supersaturation on the neutral-stability curve, which corresponds to the stationary state with crystallization at a constant nucleation rate. Note that $u_s = u_0$ in the stability region. On passage through the neutral curve, a second periodic state branches from the stationary one. The value of u_s on this curve satisfies the following stationary equation implied by (4):

$$\frac{4\pi\rho}{b}\left(\frac{b\beta c_0 u_s}{\gamma}\right)^{3/b}\int\limits_0^\infty J(u_s)\,\tau^{-1+3/b}e^{-\tau}d\tau = Q. \tag{5}$$

The same relationship applies also for stationary conditions with constant nucleation rates within the stability region. In the instability region, in general, $u_0 = u_s(1 + \langle\xi\rangle)$, i.e., for $\langle\xi\rangle \neq 0$ we have $u_s \neq u_0$. We substitute $u = u_s(1 + \xi)$ and $u_0 = u_s(1 + \langle\xi\rangle)$ into (4), use (3) and (5), and retain terms up to the third order in powers of the small quantity ξ to get the following equation:

$$\begin{gathered} d\xi/d\theta + R\xi + g_1J_1(\xi) + R\langle\xi\rangle(-1 + 3/b) + \\ + g_1\xi J_1(\xi) + g_2J_2(\xi) + R(-1 + 3/b)\langle\xi\rangle\xi + \\ + g_1(-1 + 3/b)\langle\xi\rangle J_1(\xi) + g_2\xi J_2(\xi) + g_3J_3(\xi) = 0. \end{gathered} \tag{6}$$

Here we have introduced the parameters

$$R = \frac{Q}{u_s\gamma c_0}, \quad g_k = \frac{RJ^{(k)}(u_s)\,u_s^k}{k!J(u_s)}, \quad J^{(k)} = \frac{d^kJ}{du^k}, \tag{7}$$

$$J_k(\xi) = \Gamma^{-1}(3/b)\int\limits_0^\infty \xi^k|_{\theta-\tau}\,\tau^{-1+3/b}e^{-\tau}d\tau, \quad k = 1,\ 2,\ 3,$$

where $\Gamma(x)$ is Euler's gamma function.

It is necessary to include terms up to the third order of smallness in considering oscillations even for very small but finite amplitudes, as is implied by the general theory [14]; in deriving (6), we have used the fact that $\langle\xi\rangle$ is of the order of $|\xi|^2$ according to this theory. The approximation to (6) linear in ξ enables one to determine the bounds of the stability region for homogeneous crystallization with constant crystallization rate as in [1]. We substitute the linear equation $\xi = \xi_0 \exp(\nu\theta)$ following from (6) to get the complex equation

$$\nu + R + g_1(1 + \nu)^{-3/b} = 0. \tag{8}$$

With $\nu = i\omega$, where ω is a real number, we get from (8) two real equations for the neutral-stability curve:

$$R\,\mathrm{tg}\,(3\varphi/b) = -\,\mathrm{tg}\,\varphi, \quad R = -\,g_1\cos^{3/b}\varphi\cos(3\varphi/b), \quad \varphi = \mathrm{arctg}\,\omega. \tag{9}$$

System (9) has the root $\varphi = 0$; the corresponding instability condition

$$J'(u_s)\,u_s/J(u_s) < -1 \tag{10}$$

can be satisfied on descending branches of $J(u)$ as occurring when the Tamman effect is present. In that case, stability is lost in relation to zero-frequency perturbations, and it can be shown that this leads to a supersaturation explosion, which increases without limit under given external conditions. It is impossible for self-excited oscillations to occur under these conditions.

If $b < 3$ (this is so virtually always), it follows from (9) that the system will be unstable even when

$$G = \frac{u_s}{J(u_s)}\left.\frac{dJ}{du}\right|_{u=u_s} > S(R) = (-\cos^{-3/b}\varphi)\cos^{-1}(3\varphi/b). \tag{11}$$

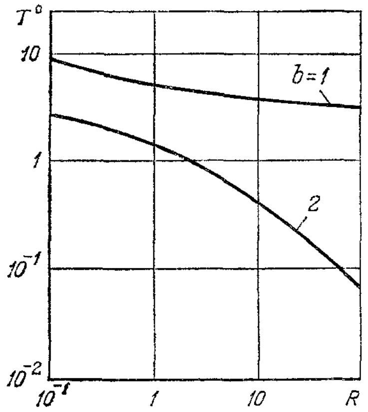

Fig. 2

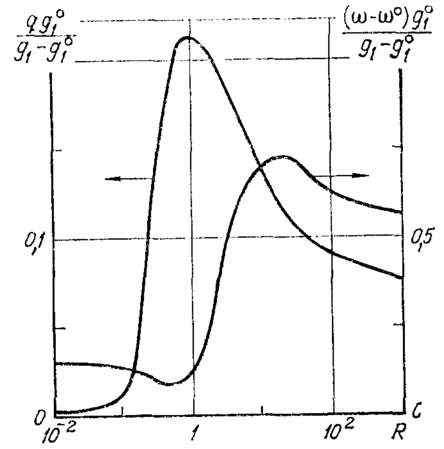

Fig. 3

Fig. 2. Dependence on R for the oscillation period on the neutral-stability curve.

Fig. 3. Dependence of the square of the oscillation amplitude and the frequency shift on R and the supercriticality $g_1/g_1^0 - 1$ in kinetic growth (b = 1) and with power-law nucleation kinetics ($J = Cu^n$).

In this case, the stability is initially perturbed in relation to small perturbations with the dimensionless frequency $\omega^0 = \mathrm{tg}\,\varphi(R)$, where the angle φ varies from $\pi b/6$ to $\min\{\pi b/3, \pi/2\}$ as R increases from zero to infinity. Figure 1 shows the neutral-stability curves corresponding to (11) together with S(1) = min S(R). In particular, if we use the power-law approximation $J(u) = Cu^n$, condition (11) becomes n > S(R). As n takes a value in the range 1 to 8 [15] usually in experiments on the rising branch of J(u), Fig. 1 shows that instability may occur over a wide range in R if b is not too large. Here we may note that an unsound conclusion was drawn in [16, 17] that oscillations are impossible with the power-law dependence of the nucleation rate on the supersaturation. Figure 2 shows how the period $T^0 = 2\pi/\omega^0$ of the small perturbations involved in the stability loss on R, which agrees with the results of [9].

For b > 3, system (9) has only the single root $\varphi = 0$ i.e., only the instability condition (10) is obeyed, not (11). This corresponds physically to the crystal growth rate increasing so slowly with the size that the stability cannot be violated even if the steepness of the ascending branch of J(u) is very great. However, values b > 3 are somewhat exotic, and if they can occur in practice at all, they must require extremely special conditions.

We assume that weakly linear almost harmonic oscillations occur with a small relative amplitude when (11) is obeyed and there is slight penetration into the instability region. We put

$$\xi(\theta) = \sum_{n=-\infty}^{\infty} \Phi_n e^{in\omega\theta}, \quad \Phi_n = \Phi_{-n}^*, \quad \Phi_1\Phi_{-1} = \Phi_1^2 = q, \tag{12}$$

where the asterisk denotes the complex conjugate, while the origin for the dimensionless time θ is taken such that Φ_1 is real. Also, $\Phi_0 = \langle\xi\rangle \sim q$, $|\Phi_n| \sim q^{n/2}$, where q is proportional to the supercriticality $g_1/g_1^0 - 1$, in which $g_1^0 = RS(R)$ is the value of g_1 on the neutral-stability curve [14]. This shows that we need to consider only terms up to order $q^{3/2}$ inclusive in (6), while in (12) we consider only harmonics having $|n| \leqslant 2$. We substitute (12) into (6) and separate the terms containing different phase factors for n = 0, 1, 2 to get the equations

$$\begin{gathered}
[g_1 + (3/b)R]\Phi_0 = -2[g_1 \operatorname{Re} A(\omega) + g_2]q, \\
[2i\omega + R + g_1 A(2\omega)]\Phi_2 = -[g_1 A(\omega) + g_2 A(2\omega)]q, \\
A(x) = (1 + ix)^{-3/b}, \\
i\omega + g_1 A(\omega) + R - (1 - 3/b)R\Phi_0 + (3/b)g_1\Phi_0 A(\omega) +
\end{gathered} \tag{13}$$

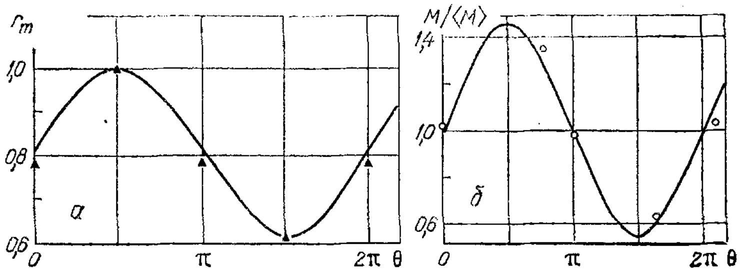

Fig. 4. Comparison of theory with experiment: a) mean crystal radius r_m (10^{-3} m) for sodium nitrate; points from experiment [19], solid line from (26); $R = 1$, $u_s = 1$, $q^{1/2} = 0.06$, $\gamma = 4.5\cdot10^{-4}$ sec^{-1}, $c_0\beta = 5\cdot10^{-4}$ m/sec, $b = 1$, $J = Cu^n$; b) crystal yield $M(\theta)/\langle M\rangle$ for sodium thiosulfate; points from experiment [19], solid line from (26); $R = 1$, $q^{1/2} = 0.075$, $\gamma = 3\cdot10^{-4}$ sec^{-1}, $b = 1$, $J = Cu^n$.

$$+ g_1\Phi_0 + g_1[A(2\omega) + A^*(\omega)]\Phi_2 + 2g_2A(\omega)(\Phi_0 + \Phi_2)$$
$$+ 2g_2q + g_2A(2\omega)q + 3g_3A(\omega)q = 0. \tag{14}$$

Without exceeding the accuracy of the theory, we can replace ω and g_1 by their values ω^0 and g_1^0 on the neutral-stability curve in terms of order q. Then (13) implies that

$$\Phi_0 = \alpha_0 q, \quad \Phi_2 = \alpha_2 q, \tag{15}$$

where we have introduced the coefficients

$$\alpha_0 = \frac{2(R - g_2)}{(3/b)R + g_1^0}, \quad \alpha_2 = \frac{i\omega^0 + R - g_2A(2\omega^0)}{2i\omega^0 + R + g_1^0A(2\omega^0)}. \tag{16}$$

We expand the first two terms in (14) as series in the small quantities $\omega - \omega^0 \sim q$ and $g_1/g_1^0 - 1 \sim q$, and use (9) to get

$$i\omega + g_1A(\omega) + R = -(g_1 - g_1^0)(R + i\omega^0)/g_1^0 + (\omega - \omega^0)(l_1 + i\,l_2), \tag{17}$$

where

$$l_1 = 3(R - 1)\omega^0/b(1 + \omega^{0^2}), \quad l_2 = 1 + 3(R + \omega^{0^2})/b(1 + \omega^{0^2}). \tag{18}$$

The complex equation (14) is equivalent to two real equations, which are used in determining the unknowns q and the frequency shift $\omega - \omega^0$. We substitute (17) and (15) into (14) and solve the linear system for q and $\omega - \omega^0$ to get

$$q = \frac{g_1 - g_1^0}{g_1^0}\,\frac{Rl_2 - \omega^0 l_1}{m_1l_2 - m_2l_1}, \tag{19}$$

$$\omega - \omega^0 = \frac{g_1 - g_1^0}{g_1^0}\,\frac{m_2R - m_1\omega^0}{m_1l_2 - m_2l_1}. \tag{20}$$

Here we have introduced the parameters

$$m_1 = (g_1^0 - R)\alpha_0 + 2g_2 - (2g_2\alpha_0 + 3g_3)R/g_1^0 + \mathrm{Re}\{g_1^0\alpha_2[A(2\omega^0) + A^*(\omega^0)] + g_2A(2\omega^0) + 2g_2\alpha_2A(\omega^0)\},$$

$$m_2 = -\omega^0(3\alpha_0/b + 2g_2\alpha_0/g_1^0 + 3g_3/g_1^0) + \mathrm{Im}\{g_1^0\alpha_2[A(2\omega^0) + A^*(\omega^0)] + g_2A(2\omega^0) + 2g_2\alpha_2A(\omega^0)\}.$$

Equations (19) and (20) enable one to determine the character of the stability loss, the conditions for the existence of weakly linear almost harmonic crystallization, and the change in the oscillation frequency with the supercriticality for any nucleation kinetics, which may take a fairly complicated form [18]. The case $q > 0$ corresponds to the normal bifurcation in the stationary state (mild instability excitation) and the formation of a secondary periodic crystallization mode, while $q < 0$ corresponds to hard excitation and oscillations with wide ranges in frequency and initial phase, whose amplitude changes stepwise from zero to a certain finite value on passing through the boundary into the instability region. As the expression in the numerator in (19) is positive (as can be verified directly), the mildness condition is equivalent to

$$m_1 l_2 > m_2 l_1, \tag{22}$$

where the quantities l_1, l_2 and m_1, m_2 are defined in (18) and (21), respectively. Then (22) and the condition $q \ll 1$, which is obeyed for sufficiently small $g_1/g_1^\circ - 1$, are sufficient to produce oscillation in homogeneous crystallization, the value of the supercriticality thus indicating only slight penetration into the instability region.

To obtain further information from (19), we need data on the nucleation kinetics. Experiment showed that $J(u)$ should be based on $J = Cu^n$ [10] or a relationship implied by the thermodynamic theory [18] in accordance with the particular conditions of mass crystallization. The results below relate to situations in which the nucleation kinetics may be described by one of these equations.

In the case $J = Cu^n$, (7) gives

$$\begin{gathered} g_1 = nR, \quad g_2 = S(R)[S(R)-1]R/2 + O(g_1/g_1^0 - 1), \\ g_3 = S(R)[S(R)-1][S(R)-2]R/6 + O(g_1/g_1^0 - 1), \end{gathered} \tag{23}$$

where $O(g_1/g_1^\circ - 1)$ is of the order of q. We neglect terms of the order of q in (23), which does not lead to accuracy loss in the approximation, and substitute (23) into (19)-(21) to get equations for the oscillation characteristics, with all the parameters dependent only on b and R. These equations are cumbersome and therefore are not given.

Figure 3 shows numerical results for $b = 1$. The stability violation is of the mild type throughout the range in R because $q > 0$. The frequency increases with the supercriticality.

The asymptotes $R \ll 1$ and $R \gg 1$ have been examined for diffusion-limited growth ($b = 2$), which showed that in these cases the oscillations are mild.

The nucleation kinetics in supersaturated solutions may be described by the following derived from the thermodynamic theory:

$$J = C \exp[-A/\ln^2(1+u)]. \tag{24}$$

We use (7) and (24) to calculate g_2 and g_3; for u_s small we have

$$\begin{gathered} g_1 = 2RA/u_s^2, \quad g_2 = S(R)[S(R)-3]R/2 + O(g_1/g_1^0 - 1), \\ g_3 = [S(R)^2 - 9S(R) + 12]S(R)R/6 + O(g_1 g_1^0 - 1). \end{gathered} \tag{25}$$

If u_s is arbitrary, the expressions for g_2 and g_3 corresponding to (24) are fairly cumbersome and are not given here. A study of the asymptotes $R \ll 1$ and $R \gg 1$ by the use of (25) was performed for the kinetic and diffusion-limited states, which showed that $q > 0$ in these cases and that near-harmonic oscillations arise near the neutral-stability curve.

Supersaturation oscillations alter the mean crystal radius, the yield of finished crystals, and so on [5, 6, 17, 19]. In principle, the theory enables one to describe all these effects. For example, in the kinetic state, the dependence of the mean radius r_m and the crystal yield M on time takes the form

$$\begin{aligned} r_m = \int_0^\infty rf(t, r)\,dr = \frac{\gamma^2}{8\pi u_s \rho \beta^2 c_0^2} \{ & R - 2R\Phi_0 + (6R - 2\omega^2)q \\ & + (2\omega^2 - 4R)q^{1/2}\cos\omega\theta + 2(1+R)\omega q^{1/2}\sin\omega\theta + \\ & + [4R\,\mathrm{Im}\,\Phi_2 + 4\omega(1+R)\,\mathrm{Re}\,\Phi_2 - 4(1+R)\omega q - 8\omega^2\,\mathrm{Im}\,\Phi_2]\sin 2\omega\theta \\ & + [6(R-\omega^2)q - 4R\,\mathrm{Re}\,\Phi_2 + 4(1+R)\omega\,\mathrm{Im}\,\Phi_2 + 8\omega^2\,\mathrm{Re}\,\Phi_2]\cos 2\omega\theta\}, \end{aligned}$$

$$\begin{aligned} M = \gamma\rho V \int_0^\infty f\,dr = \frac{\gamma^2 \rho V}{\beta u_s c_0}\left(r_m + \frac{dr_m}{d\theta}\right) & [1 - \Phi_0 + 2q - 2q^{1/2}\cos\omega\theta \\ & - 2(\mathrm{Re}\,\Phi_2 - q)\cos 2\omega\theta + 2\,\mathrm{Im}\,\Phi_2 \sin 2\omega\theta] + O(q^{3/2}), \quad O(q^{3/2}) \sim q^{3/2}. \end{aligned} \tag{26}$$

Figure 4 compares the results from (26) with the experimental data of [19]. We see from (15) and (26) that for $g_2 > R$ the mean yield $\langle M \rangle$ should increase in proportion to $g_1/g_1^\circ - 1$ and can attain several percent even for low supercriticality. This has been observed by experiment [16, 17].

The results show that the proposed theory gives an adequate description of mass crystallization in an ideal-mixing system with continuous solid-particle removal, and it can be used to examine the processes in such equipments. It is readily seen that this theory can be extended to more complicated situations without essential difficulty, such as when the external mass flux varies, the various effective parameters are dependent on the supersaturation, and so on.

In conclusion, we return to the interpretation of [1] for the boiling model and note that $G = J'u_s/J \sim 10^2\text{-}10^3$ for boiling [20]. Figure 1 shows that in the kinetic state of bubble growth, this value of G corresponds to deep penetration into the instability region. The oscillation amplitude in that case is usually large, and the oscillations themselves are far from harmonic. It is therefore undesirable to use this theory of weakly nonlinear oscillations under the conditions of homogeneous boiling.

NOTATION

A, C, n, constants specifying nucleation kinetics; b, parameter introduced in (3); c, c_0, concentration, saturation concentration; D, diffusion coefficient; f, function of crystal size distribution; J, $J^{(k)}$, nucleation rate and its derivative of the k-th order; J_k, integral defined in (7); l_1, l_2, parameters introduced in (18); M, crystal yield; m_1, m_2, parameters introduced in (21); r, r_*, r_m, crystal radius, critical nucleus radius, and mean crystal radius, respectively; R, parameter introduced in (7); q, square of the amplitude of the fundamental of the supersaturation perturbation; Q, mass flow; S, G, functions defined in (11); t, time; u, dimensionless supersaturation; V, crystallizer volume; v, crystal volume; α, index in the equation for crystal growth rate; α_0, α_2, functions defined in (16); β, kinetic coefficient of crystal growth rate; γ, mass-transfer coefficient; θ, dimensionless time; ν, parameter introduced in (8); ξ, supersaturation perturbation; ρ, crystal density; φ, parameter specified in (9); Φ_n, amplitude of the n-th harmonic of supersaturation perturbation; ω, frequency; a superscript degree refers to quantities defined on the neutral stability curve; an asterisk indicates complex conjugation; angle brackets imply time averaging.

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MEASUREMENT OF THE FLUID FLOW VELOCITY BY AN ACOUSTIC FREQUENCY—PHASE METHOD

O. G. Martynenko, V. I. Krylovich,
A. D. Solodukhin, and V. G. Fedorei

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New frequency—phase acoustic methods are described as well as the principles for apparatus realization of the recording the flow parameters of a fluid (sea) medium possessing elevated sensitivity and low inertia.

The most extensively used devices at the present time for the determination of the mass-transfer characteristics, for instance, the flow velocities of fluid (marine) media, are vane-type devices based on a mechanical method of measurement. The possibilities of these traditional apparatus with respect to sensitivity, accuracy, and time resolution are restricted and practically exhausted. Thus, for example, the accuracy of measuring the flow velocity of one of the last serially manufactured devices of the type ATsIT [1] with a vane-type flow velocity sensor does not exceed 3-4 cm/sec.

To obtain the background characteristics of a marine medium, to investigate its fine structure, and to study the energy exchange between the ocean and the atmosphere, the circulation of vortex flows, the wake of typhoons, etc., the accuracy and sensitivity of the flow velocity measurements must be raised substantially and the sensor inertia must be diminished.

Most promising in these respects are acoustic methods. When they are used, the flow velocity is determined by the difference in the times of ultrasonic signal (pulse) passage with and against the flow in the liquid (gaseous) medium being investigated, or the phase shift of the received and emitted ultrasonic oscillations caused by the motion of the medium, or by the magnitude of the Doppler effect that occurs during reflection of the ultrasonic wave from the inhomogeneities of the moving medium [1, 2]. However, these methods do not afford the possibility of simultaneous measurement of the magnitude of the flow velocity and acceleration.

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A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Institute of Applied Physics, Academy of Sciences of the Belorussian SSR, Minsk. Far-East Scientific-Research Institute, Goskomgidromet SSSR, Vladivostok. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 2, pp. 242-252, August, 1985. Original article submitted September 6, 1984.

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